Simulating the Strong Force with Neural Networks

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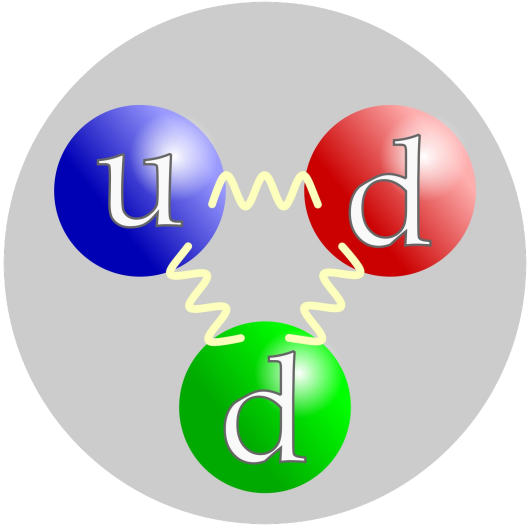
What is Strong Force?

Four Fundamental Forces:

- -Gravity
- -E/M
- -Weak Force
- -Strong Force

Strong Force holds together quarks inside protons and neutrons



Neutron

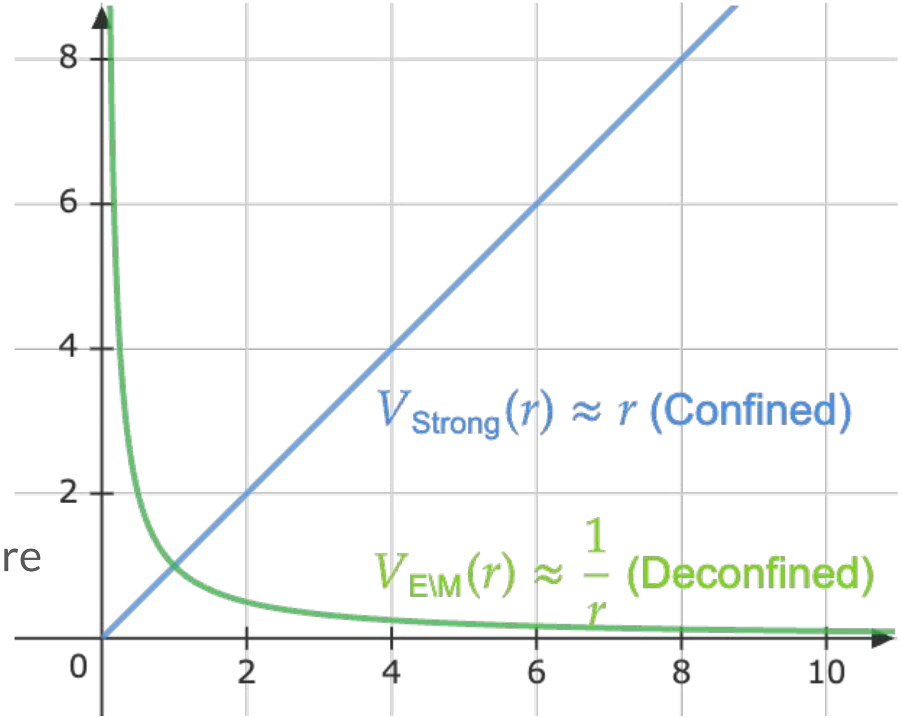
Purpose

-Strong Force is weird

-Strengthens with distance (r)

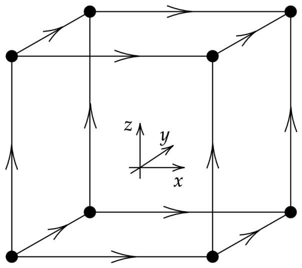
-Calculations hard

-Current numerical methods require supercomputers



Toy Model: Z_2 Gauge Theory

Simpler model that captures behavior of strong force



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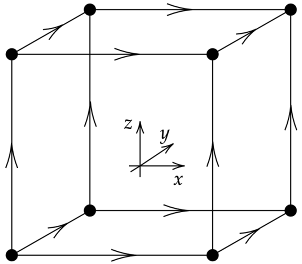

Quantum State: $|\psi\rangle = |01\rangle$

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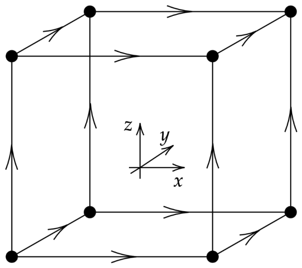

Quantum State: $|\psi\rangle = |01\rangle$

$$H = H_E + H_B \approx g^2 \sum_{l \in \text{links}} [2 - P_l] + \frac{1}{g^2} \sum_{\substack{(l_1,l_2,l_3,l_4) \\ \in \text{plaqs}}} \left[2 - Q_1^\dagger Q_2^\dagger Q_3 Q_4\right]$$

Toy Model: Z_2 Gauge Theory

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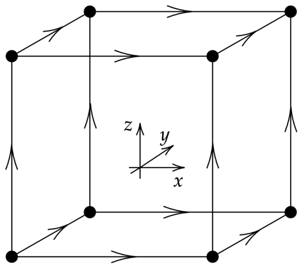

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Find minimum energy state: $E_0 = \langle \psi_0 | H | \psi_0 \rangle$

[Horn, Weinstein, Yankielowicz ‘79]

Problem: The world is not a 2x2x2 lattice

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = |01\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix} 1\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 0\begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Complications in the Math: $|\psi\rangle$ cannot be solved

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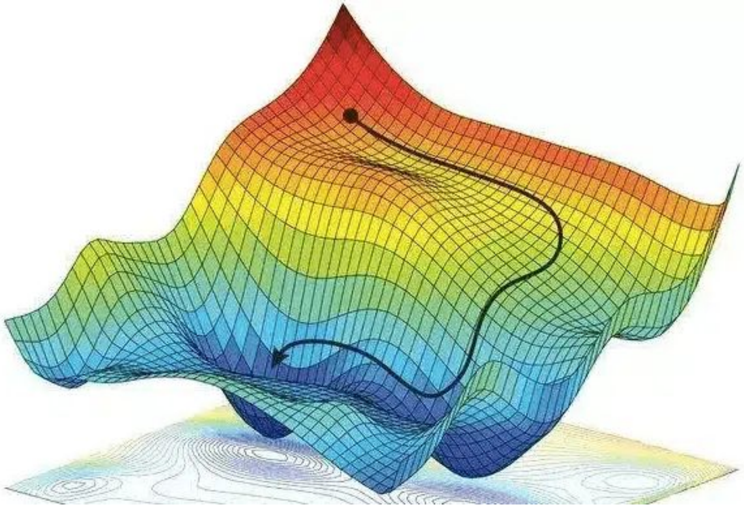
$$dim(\psi) = 2^{\#\text{links}} = 2^{3(\#\text{nodes})} = 2^{3n^3}$$

E.g. 6x6x6 ->1.17x10^195

Complications in the Math: $|\psi\rangle$ cannot be solved

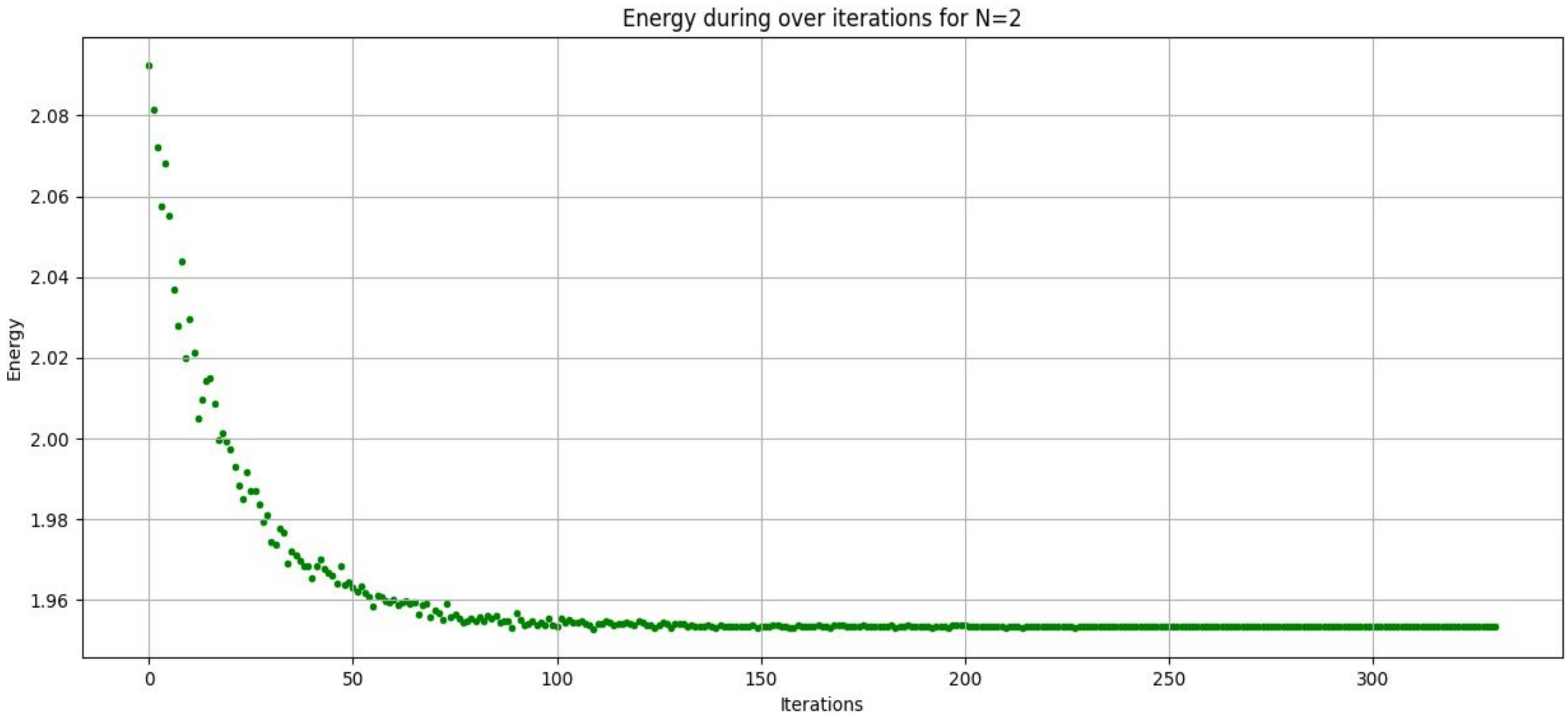


Machine Learning Solution

NN Quantum State

State $\|\psi\rangle$	$\rightarrow$ NN
Energy	$\rightarrow$ Loss Function
Finding $\|\psi_0\rangle$	$\rightarrow$ Training

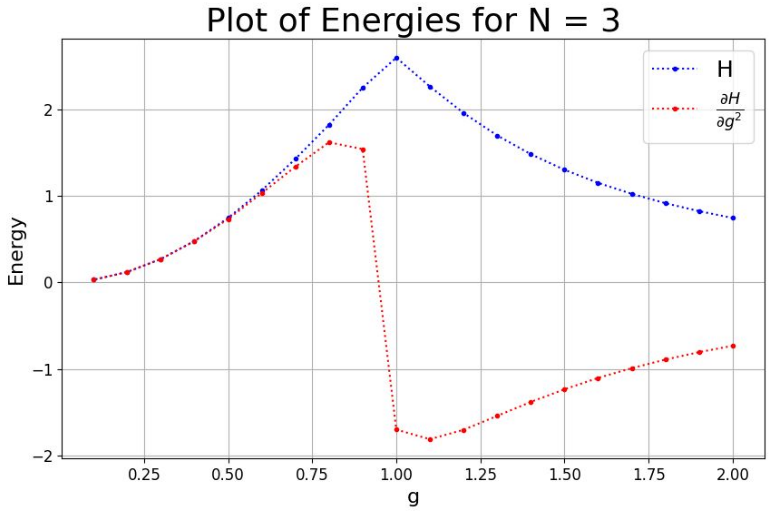
[Carleo, Troyer '16; Favoni et al. '21; Ashmore et al. '24]


Energy during over iterations for N=2
Energy
2.08
2.06
2.04
2.02
2.00
1.98
1.96
Iterations
0
50
100
150
200
250
300

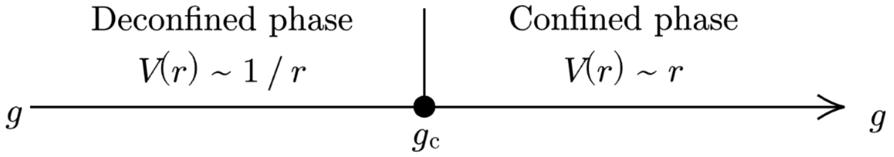
Now what?

What do we do with $|\psi_0\rangle$?

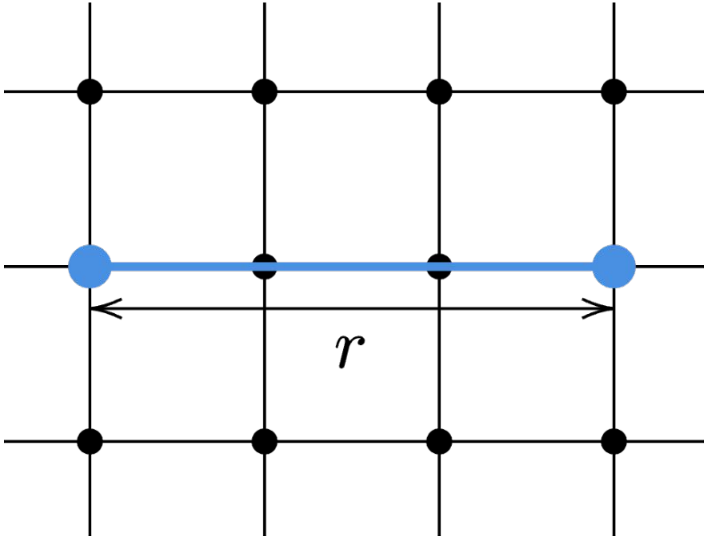
Critical Coupling Constant (g_c)

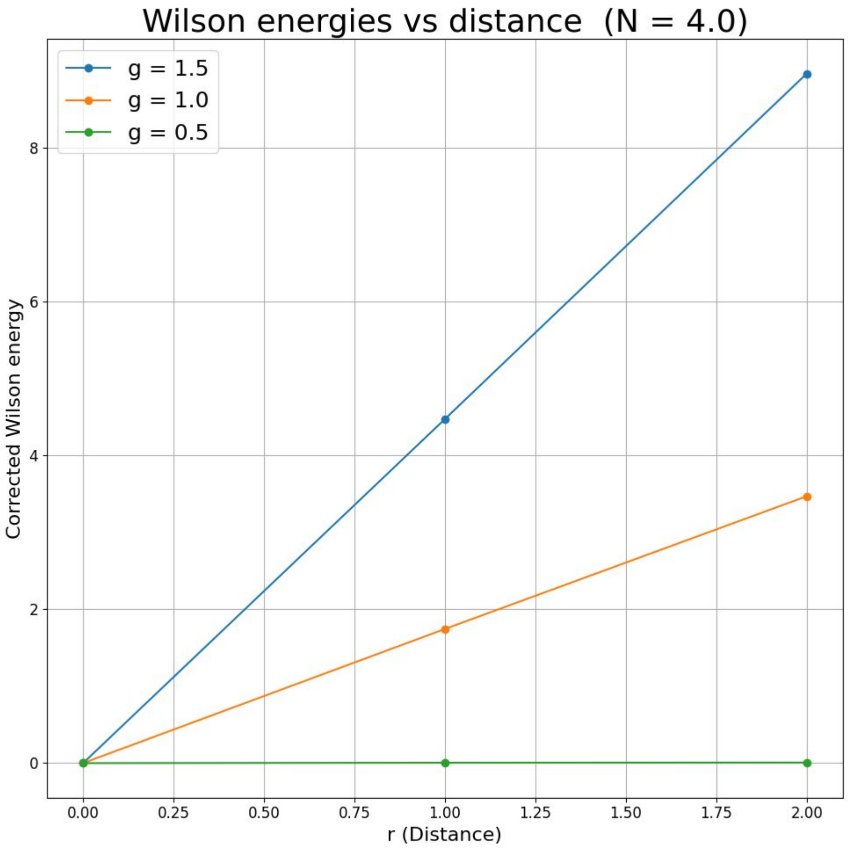


Varying g gives a first order phase transition
E.g. water → steam



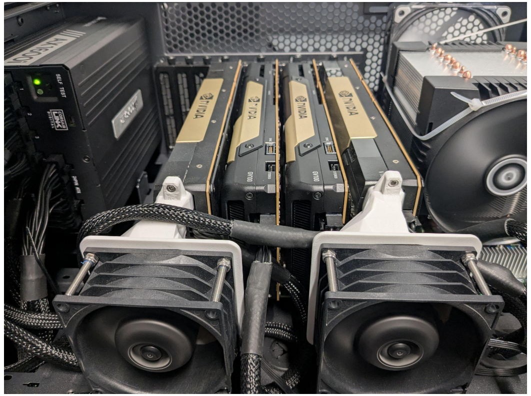
Test Charges





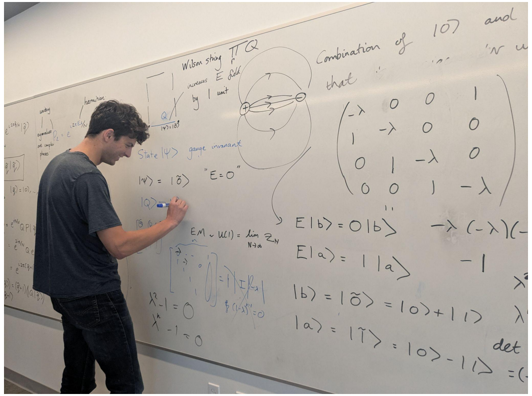
Conclusion

Using NN to model the strong force is a promising method.

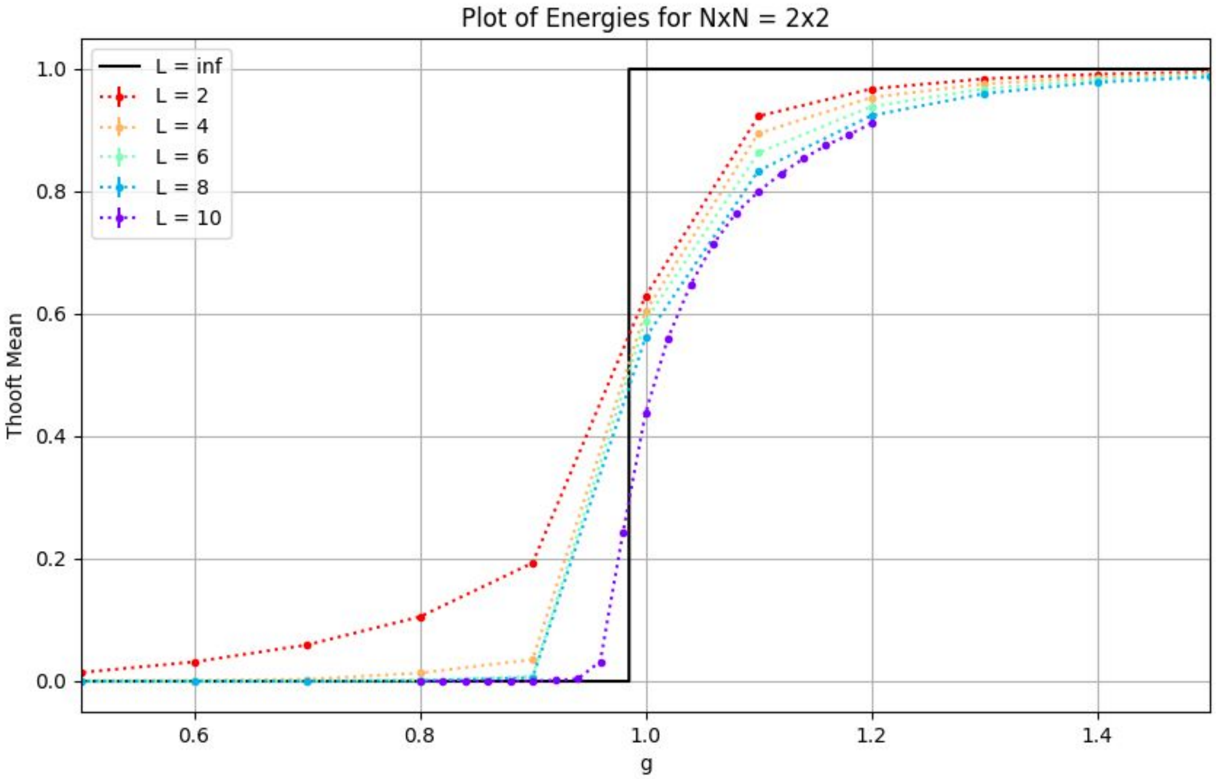


Future Work

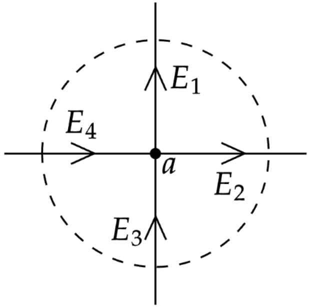
LxNxN rectangular lattices to calculate g_c



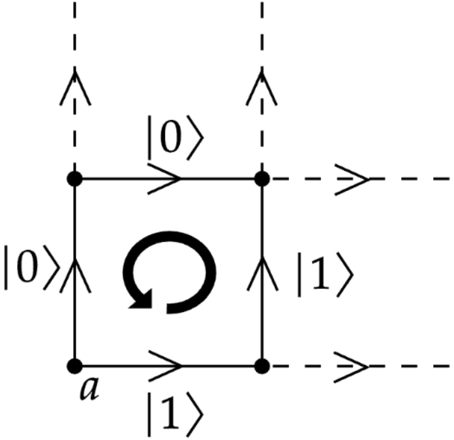
LxNxN Findings



$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + \ldots = \sum_c \psi(c)|c\rangle$$



$$\begin{aligned} Gauss:\ 0 &= E_1 + E_2 - E_3 - E_4 \\ 1 &= e^{i\pi(E_1+E_2-E_3-E_4)} \\ &= P_1 P_2 P_3^\dagger P_4^\dagger = \Theta_a \\ |\psi\rangle &= \Theta_a|\psi\rangle \end{aligned}$$



$$plaq_a = Q_1 Q_2 Q_3^\dagger Q_4^\dagger = (-)(-)(+)(+) = 1$$

Extra Equations